MATHEMATICS GRADE 12 MHF4U, ADVANCED FUNCTIONS **TEST Chapter 1** **INEQUATIONS, EQUATIONS, AND GRAPHS** Teacher: Teodoru Gugoiu	Date WINTER 2009 Name SOLUTIONS	$\frac{\quad}{50}$ =

Questions 1-5 are Multiple-Choice questions [K/U 1 mark each]

1. Which of the following relations is NOT true?

A) $|a-b|=|b-a|$ B) $\sqrt{a^2}=|a|$ C) $|a|\geq 0$ (D) $|-2a|<2|-a|$ E) $|a-b|\leq|a|+|b|$

2. The interval notation for $-3\leq x<2$ is;

A) $(-3,2)$ B) $[-3,2]$ C) $[-3,3)$ D) $[-2,3)$ (E) $[-3,2)$

3. The inequality $x^2\geq 16$ is equivalent (has the same solution set) with:

A) $|x|\geq 16$ (B) $|x|\geq 4$ C) $|x|\leq 4$ D) $|x|\leq 16$ E) $|x|\geq 8$

4. The distance between the point $A(2,-3)$ and $B(-6,+3)$ is equal to:

A) $\sqrt{10}$ B) 100 C) 5 D) 20 (E) 10

5. The y-intercept point(s) of the relation $\frac{x^2}{4}+\frac{y^2}{9}=1$ is (are):

A) $(-3,0)$ *and* $(3,0)$ B) $(0,3)$ *and* $(3,0)$ (C) $(0,-3)$ *and* $(0,3)$ D) $(-3,0)$ E) $(3,0)$

Questions 6-10 are True-False questions [K/U 1 mark each]

6. If the point $P(a,-b)$ is in quadrant II, then the point $Q(\overset{+}{-a},\overset{-}{\frac{1}{b}})$ is in quadrant IV. (T) F

7. A graph is symmetric with respect to the origin O, if whenever $P(x,y)$ is a point on the graph, $P(-x,-y)$ is also a point on the graph. (T) F

8. There is a correspondence one-to-one between the ordered pairs (a,b) and the points on a plane. (T) F

9. The relation $x^2y^2=1$ is symmetric with respect to the origin O. (T) F

10. The un-bounded interval $[2,\infty)$ is open always on the right because ∞ is not considered a number. (T) F

11. Match the relations from the left side with a graph from the right side. Some functions may have no corresponding graph. [A 4 marks]

X A) $|x-1|=2$ $x-1=-2$ $x-1=2$

B) $x^2-4=5$ I)

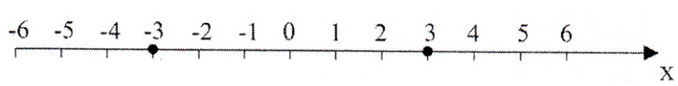
X C) $|x|>2$

D) $(x-2)(x-5)<0$ II)

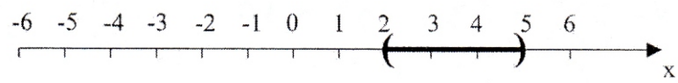
E) $\frac{1}{x}<1$ IV)

F) $\sqrt{x}>2$ III)

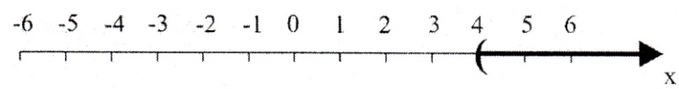
I) B



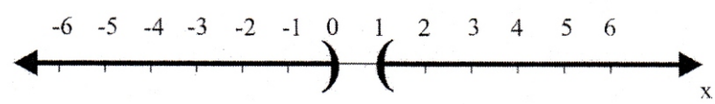
II) D



III) F



IV) E



12. Solve each inequality and then graph it. [K/U] [4 marks]

a) $2+2x \le -8+4x$

$2+8 \le 4x-2x$

$10 \le 2x$

$2x \ge 10$

$\therefore\ x \ge 5$

b) $1+2(x+3) < -3(1+2x)+4$

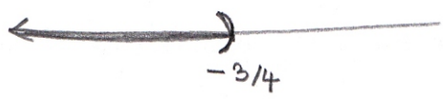
$1+2x+6 < -3-6x+4$

$2x+6x < 1-7$

$8x < -6$

$x < -\frac{6}{8}$

$\therefore\ x < -\frac{3}{4}$



13. Solve each equality or inequality and then graph it. [K/U] [4 marks]

a) $|4+3x| = 6$

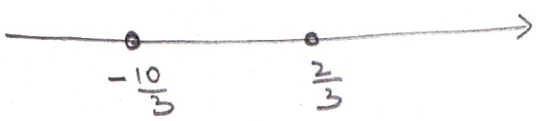
$4+3x = \pm 6$

$4+3x = 6$ OR $4+3x = -6$

$3x = 6-4$ OR $3x = -6-4$

$3x = 2$ OR $3x = -10$

$x = \frac{2}{3}$ OR $x = -\frac{10}{3}$



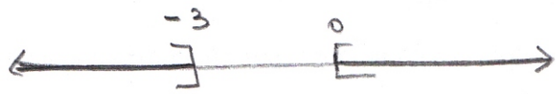
b) $|2x+3| \ge 3$

$2x+3 \ge 3$ OR $2x+3 \le -3$

$2x \ge 0$ OR $2x \le -6$

$x \ge 0$ OR $x \le -3$

$\therefore\ x \ge 0$ OR $x \le -3$



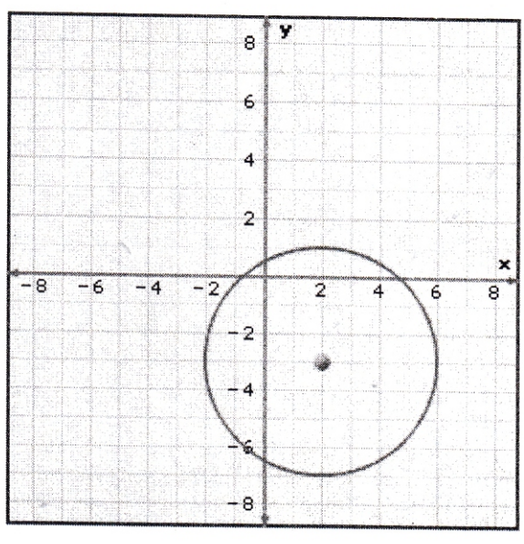
14. Write the equation of the circle represented in the right figure. [K/U] [2 marks]

$x_c = 2$

$y_c = -3$

$R = 4$

$\therefore\ (x-2)^2 + (y+3)^2 = 16$



15. Analyse the symmetry of each equation. Do not graph. [A] [6 marks]

a) $y = x^2$

$x \to -x \Rightarrow y = (-x)^2 \Rightarrow y = x^2$ (same)
[symmetry in y-axis]

$y \to -y \Rightarrow -y = x^2$ (different)
[no symmetry in x-axis]

$x \to -x$, $y \to -y \Rightarrow -y = (-x)^2 \Rightarrow -y = x^2$
(different) [no symmetry in origin]

b) $xy = x^2 + y^2$

$x \to -x \Rightarrow -xy = (-x)^2 + y^2$
$\Rightarrow -xy = x^2 + y^2$ (different)
[no symmetry in y-axis]

$y \to -y \Rightarrow x(-y) = x^2 + (-y)^2 \Rightarrow$
$-xy = x^2 + y^2$ (different)
[no symmetry in x-axis]

$x \to -x$, $y \to -y \Rightarrow (-x)(-y) = (-x)^2 + (-y)^2$
$\Rightarrow xy = x^2 + y^2$ (same)
[symmetry in origin O]

c) $x = |y|$

$x \to -x \Rightarrow -x = |y|$ (different)
[no symmetry in y-axis]

$y \to -y \Rightarrow x = |-y| \Rightarrow x = |y|$
(same) $\Rightarrow$ [symmetry in x-axis]

$x \to -x$, $y \to -y \Rightarrow -x = |-y| \Rightarrow -x = |y|$
(different)
[no symmetry in origin]

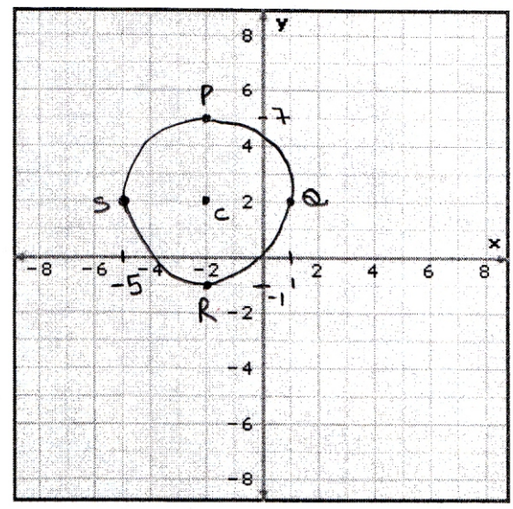
16. Graph the following circle on the grid provided on the right. [A] [4 marks]

$x(x+4) = 1 + y(4-y)$

$x^2 + 4x + y^2 - 4y = 1$

$(x+2)^2 - 4 + (y-2)^2 - 4 = 1$

$(x+2)^2 + (y-2)^2 = 3^2$



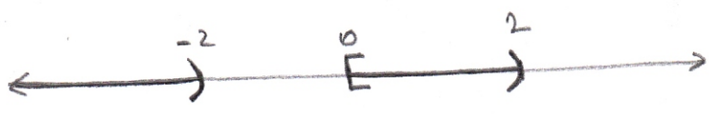
17. Solve the inequality and then graph it. Show your work. [A] [4 marks]

$$\frac{x(x-1)^2}{(x+2)^3(x-2)} \leq 0$$

$x \neq -2, 2$

X	−2	0	1	2
x	− − −	0 +	+	+ +
$(x-1)^2$	+ + +	+	0 +	+ +
$(x+2)^3$	− − −	0 +	+	+ +
$x-2$	− − −	−	−	0 + +
ALL	− − − |	+ + 0 − −	0 − − |	+ +

$\therefore \quad x < -2 \text{ or } 0 \leq x < 2 \quad ; \quad x \neq -2, 2$



18. Graph the following semicircle $y = 2 - \sqrt{4x - x^2}$. Show your work. [A] [4 marks]

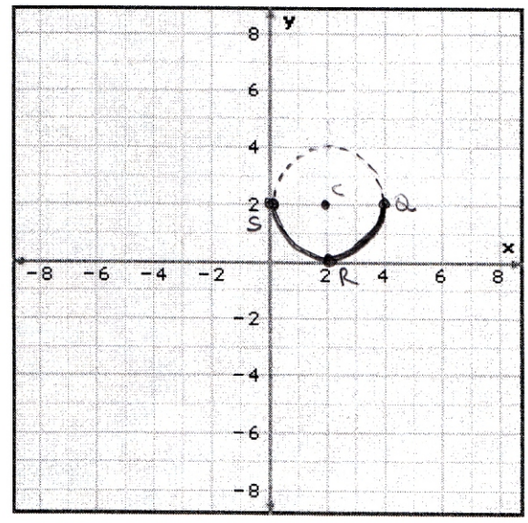
$(y-2)^2 = 4x - x^2$ $\quad y \leq 2$

$x^2 - 4x + (y-2)^2 = 0$

$(x-2)^2 + (y-2)^2 = 2^2$

$C(2,2)$

$R = 2$



19. Solve the inequality and then graph it. Show your work. [A] [5 marks]

$\frac{1}{x} - \frac{1}{x+2} \leq \frac{1}{x-1}$

$\frac{1}{x} - \frac{1}{x+2} - \frac{1}{x-1} \leq 0$

$\frac{(x+2)(x-1) - x(x-1) - x(x+2)}{x(x+2)(x-1)} \leq 0$

$\frac{x^2 + x - 2 - x^2 + x - x^2 - 2x}{x(x+2)(x-1)} \leq 0$

$\frac{-2 - x^2}{x(x+2)(x-1)} \leq 0$

$\frac{x^2 + 2}{x(x+2)(x-1)} \geqslant 0$

x	-2		0		1	
x^2+2	+	+	+	+	+	+
x	−	−	0	+	+	+
$x+2$	−	0	+	+	+	+
$x-1$	−	−	−	−	0	+
ALL	−	‖ +	0	−	‖	+

$\therefore x \in (-2, 0) \cup (1, \infty)$; $x \neq 0, -2, 1$ (checked)

20. Graph the following relation $|x| - |y| = 2$ [T/I] [3 marks]

Quadrant #1

$x > 0$, $y > 0$ $\Rightarrow$ $x - y = 2$ $\Rightarrow$ $y = x - 2$

Symmetry in x- and y-axis $\Rightarrow$