MATHEMATICS GRADE 12 MHF4U, ADVANCED FUNCTIONS **TEST Chapter 2 FUNCTIONS** Teacher: Teodoru Gugoiu	Date WINTER 2009 Name	$\overline{}$ 50 =

Questions 1-5 are Multiple-Choice questions [K/U 1 mark each]

1. Given the function $f(x)=\dfrac{2x^2}{1+x^2}$, the value of $f(-1)$ is equal to:

A) 3 B) -3 C) -2 D) 2 E) 1

2. Consider a function one-to-one $f:(-3,\infty)\to[2,3]$. The domain of f^{-1} is given by:

A) $[-2,-3]$ B) $[2,3]$ C) $(-3,\infty)$ D) $[-1,0)$ E) $[3,\infty)$

3. Consider a relation defined by a set of ordered pairs: $\{(0,1),(2,0),(-1,0),(0,0),(-1,-1)\}$. The range of this relation is:

A) $\{-1,0,1\}$ B) $\{0,1\}$ C) $\{-1,0\}$ D) $\{0,2,-1\}$ E) $(-\infty,\infty)$

4. The domain of the function defined by $f(x)=3-2\sqrt{x-1}$ is:

A) $(-\infty,\infty)$ B) $(-\infty,-2)$ C) $[1,\infty)$ D) $[-1,\infty)$ E) $(-\infty,-2]$

5. The inverse function of $f(x)=\sqrt{x-1}$ is:

A) $f^{-1}(x)=(x+1)^2$ B) $f^{-1}(x)=x^2$ C) $f^{-1}(x)=x^2-1$ D) $f^{-1}(x)=x^2+1$ E) $f^{-1}(x)=(x-1)^2$

Questions 6-10 are True-False questions [K/U 1 mark each]

6. The inverse of a one-to-one function is also a function. T F

7. The domain of the inverse function is the same as the domain of the original function. T F

8. The inverse of a radical function is also a radical function. T F

9. The function $y=-2+3\sqrt{x-1}$ is an one-to-one function. T F

10. The inverse of a linear function is [redacted] a quadratic function. T F

11. Match the functions from the left side with a graph from the right side. Some functions have no corresponding graph. [A 4 marks]

A) $f(x)=2+\sqrt{x-1}$

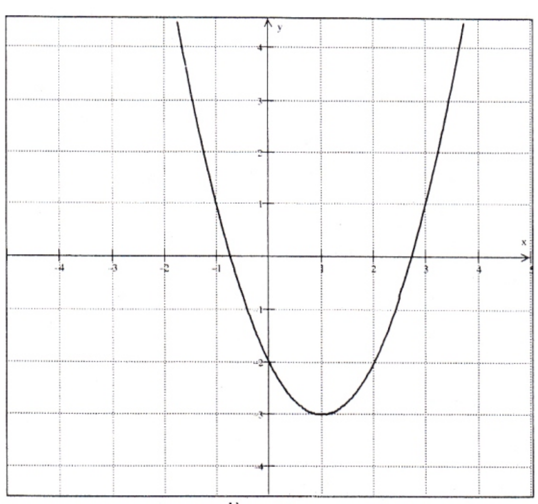
B) $g(x)=(x+1)+3$

I C) $h(x)=(x-1)^2-3$

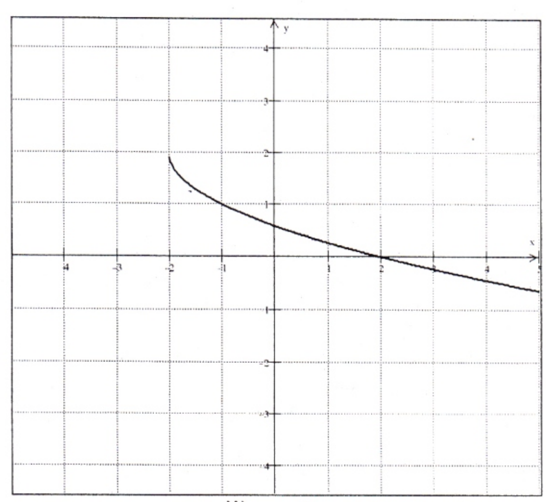
III D) $k(x)=\dfrac{x+1}{x-1}$

IV E) $p(x)=2x-3$

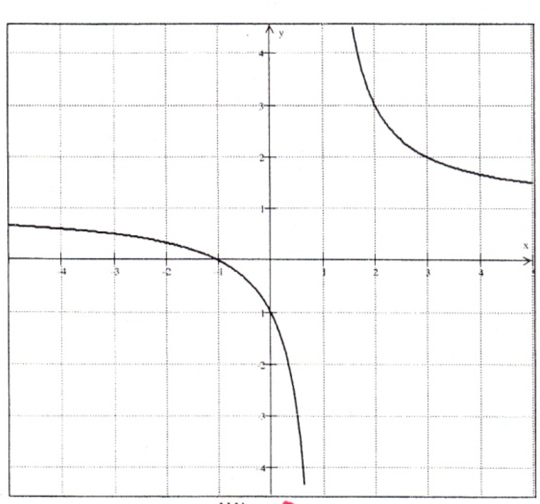
II F) $q(x)=2-\sqrt{x+2}$



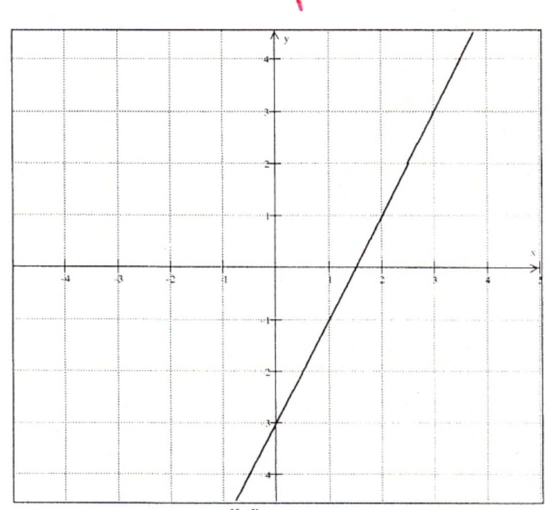
I) ..C..



II) ....F



III) ...D..



IV) ... E

The following questions are long answer questions. Show your work to get full marks.

12. Find the inverse of the following functions: [T/I 4 marks]

a) $f(x) = 2 - \sqrt{\dfrac{2x-1}{3}}$

$y = 2 - \sqrt{\dfrac{2x-1}{3}}$

$x = 2 - \sqrt{\dfrac{2y-1}{3}}$

$\sqrt{\dfrac{2y-1}{3}} = 2 - x$

$2y - 1 = 3(2-x)^2$

$y = \dfrac{3(2-x)^2+1}{2}$

$\therefore f^{-1}(x) = \dfrac{3(2-x)^2+1}{2}$

b) $f(x) = 2 - \dfrac{x}{2x-3}$

$y = 2 - \dfrac{x}{2x-3}$

$x = 2 - \dfrac{y}{2y-3}$

$\dfrac{y}{2y-3} = 2 - x$

$y = 4y - 6 - 2xy + 3x$

$y = \dfrac{3x-6}{2x-3}$

$\therefore f^{-1}(x) = \dfrac{3(x-2)}{2x-3}$

13. Classify each function as even, odd, or neither. [K/U 3 marks]

a) $f(x) = x^3 - x^2 + x - 1$

$f(-x) = (-x)^3 - (-x)^2 + (-x) - 1$

$= -x^3 - x^2 - x - 1$

$\neq f(x)$

$\neq -f(x)$

$\therefore$ neither even ~~nor~~ nor odd.

b) $f(x) = x^4 - x^2$

$f(-x) = (-x)^4 - (-x)^2$

$= x^4 - x^2$

$= f(x)$

$\therefore$ f is even

c) $f(x) = x + \dfrac{1}{x^3}$

$f(-x) = -x + \dfrac{1}{(-x)^3}$

$= -x - \dfrac{1}{x^3}$

$= -f(x)$

$\therefore$ f is odd

14. Express $f(x) = x - |x-1| + |x-2|$ as a piecewise-defined function. Do not graph. [K/U 4 marks]

$$f(x) = \begin{cases} x - (1-x) + 2 - x, & x < 1 \\ x - (x-1) + 2 - x, & 1 \le x \le 2 \\ x - (x-1) + x - 2, & x > 2 \end{cases} = \begin{cases} x + 1, & x < 1 \\ 3 - x, & 1 \le x \le 2 \\ x - 1, & x > 2 \end{cases}$$

15. Find the values of the parameter k such that the quadratic equation $2kx^2 - (2k+1)x + k = 0$ has two real distinct roots. [A 4 marks]

$\Delta = [-(2k+1)]^2 - 4(2k)(k)$

$= 4k^2 + 4k + 1 - 8k^2$

$= -4k^2 + 4k + 1$

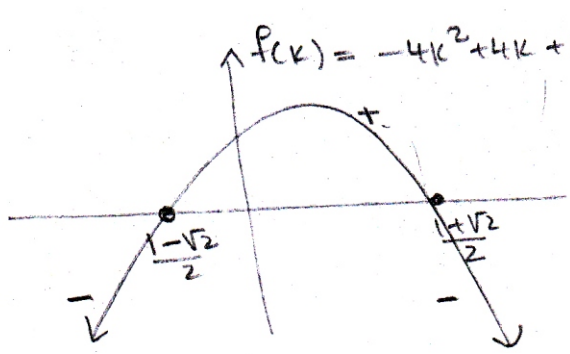
$\Delta > 0$

$-4k^2 + 4k + 1 > 0$

$k = \dfrac{-4 \pm \sqrt{16+16}}{-8}$

$= \dfrac{-4 \pm 4\sqrt{2}}{-8}$

$= \dfrac{1}{2} \pm \dfrac{\sqrt{2}}{2}$



$f(x) > 0$ if

$\therefore \dfrac{1-\sqrt{2}}{2} < k < \dfrac{1+\sqrt{2}}{2}$

& $k \neq 0$

16. Consider the function: [A 7 marks]

$f(x) = -4x^2 - 4x + 3 \quad , \quad x \geq -1/2$

[2] a) State the domain and the range of the function $f(x)$

$f(x) = -4\left(x^2 + x + \frac{1}{4} - \frac{1}{4}\right) + 3$

$= -4\left(x + \frac{1}{2}\right)^2 + 4$

$D_f = [-\frac{1}{2}, \infty) \qquad R_f = (-\infty, 4]$

x-int

$-4\left(x+\frac{1}{2}\right)^2 + 4 = 0$

$1 = \left(x + \frac{1}{2}\right)^2$

$x + \frac{1}{2} = \pm 1$

$x = -\frac{1}{2} \pm 1$

[2] b) Find the inverse function $f^{-1}(x)$

$x = -4\left(y + \frac{1}{2}\right)^2 + 4$

$4\left(y + \frac{1}{2}\right)^2 = 4 - x$

$y + \frac{1}{2} = \pm\sqrt{\frac{4-x}{4}}$

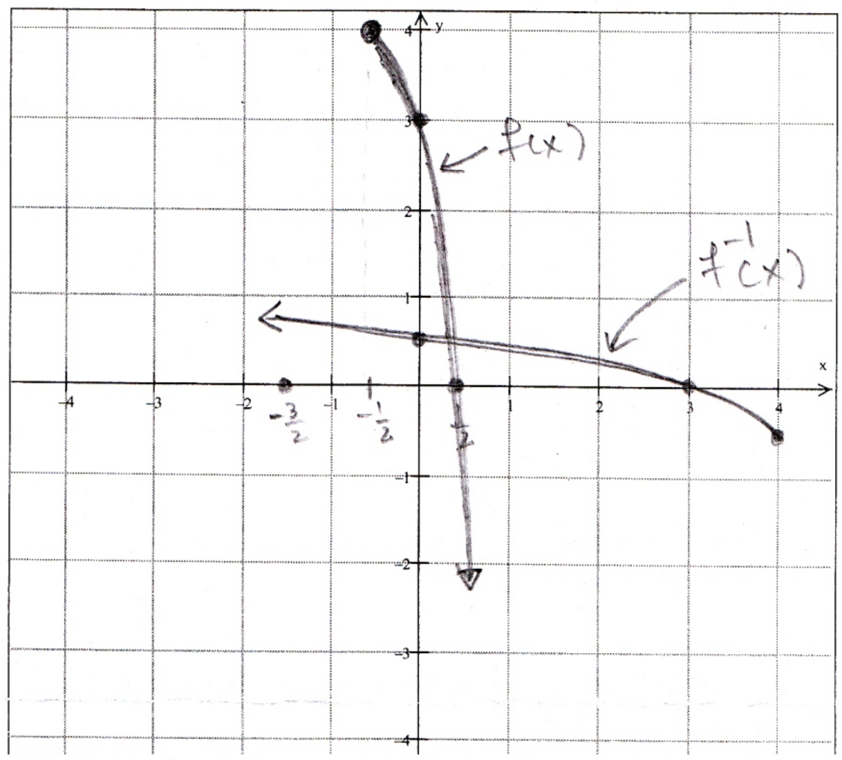
$\therefore f^{-1}(x) = -\frac{1}{2} + \sqrt{\frac{4-x}{4}}$

[1] c) State the domain and the range of the function $f^{-1}(x)$

$D_{f^{-1}} = (-\infty, 4]$

$R_{f^{-1}} = [-\frac{1}{2}, \infty)$

[2] d) Sketch the graph of the functions $f(x)$ and $f^{-1}(x)$ on the grid provided on the right



17. Find the points of intersection between the line $7y - x - 25 = 0$ and the circle $x^2 + y^2 = 25$. Graph the line and the circle on the grid provided. [A 5 marks]

$\begin{cases} x^2 + y^2 = 25 \\ x = 7y - 25 \end{cases}$

$y^2 + (7y - 25)^2 = 25$

$50y^2 - 350y + 25^2 = 25$

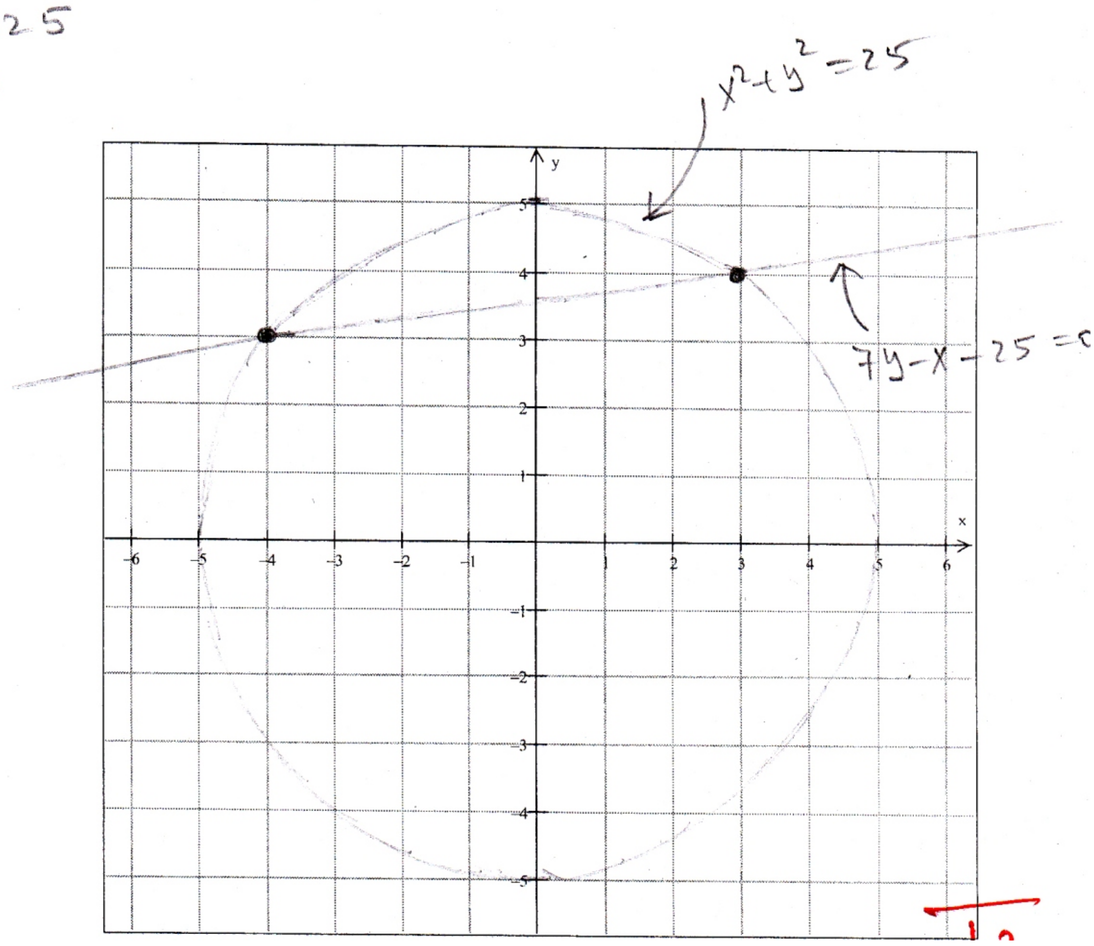
$2y^2 - 14y + 24 = 0$

$y^2 - 7y + 12 = 0$

$(y-3)(y-4) = 0$

$\begin{cases} y = 3 \\ x = 7 \times 3 - 25 = -4 \end{cases}$

$\begin{cases} y = 4 \\ x = 7 \times 4 - 25 = 3 \end{cases}$



18. Consider the following functions: $f(x)=(x-2)^2+1$ and $g(x)=2-\sqrt{1-x}$. [A 6 marks]

[2] a) Find the function $(f\circ g)(x)$.

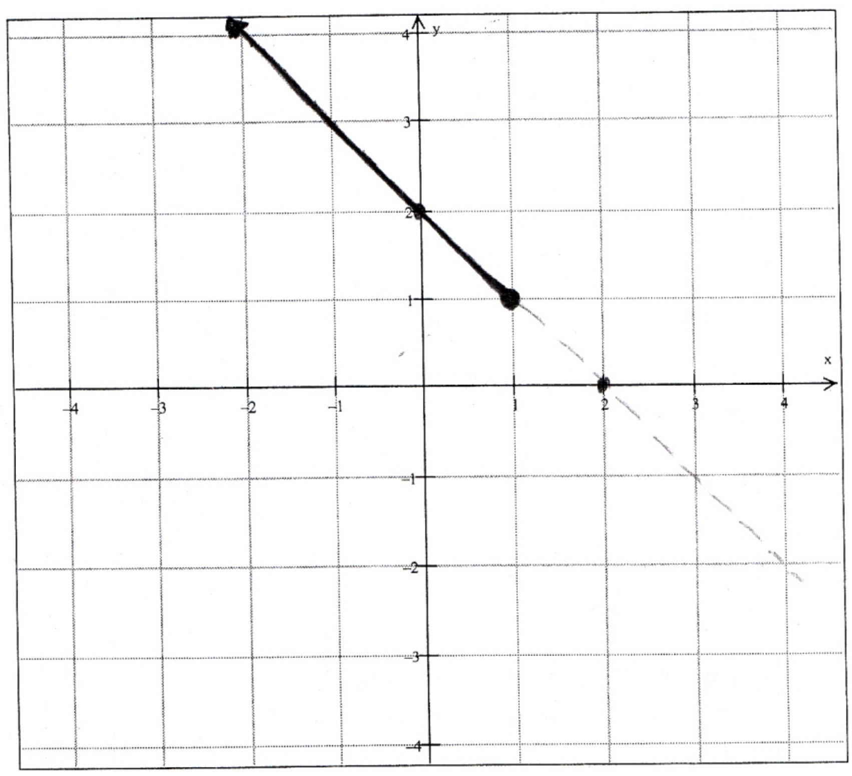
$f(g(x)) = f(2-\sqrt{1-x}) = (\cancel{2}-\sqrt{1-x}-\cancel{2})^2+1 = 1-x+1 = 2-x$

[2] Find the domain and the range of the composition $f\circ g$.

$(-\infty, 1] \xrightarrow{g(x)=2-\sqrt{1-x}} (-\infty, 2] \xrightarrow{f(x)=(x-2)^2+1} [1,\infty)$

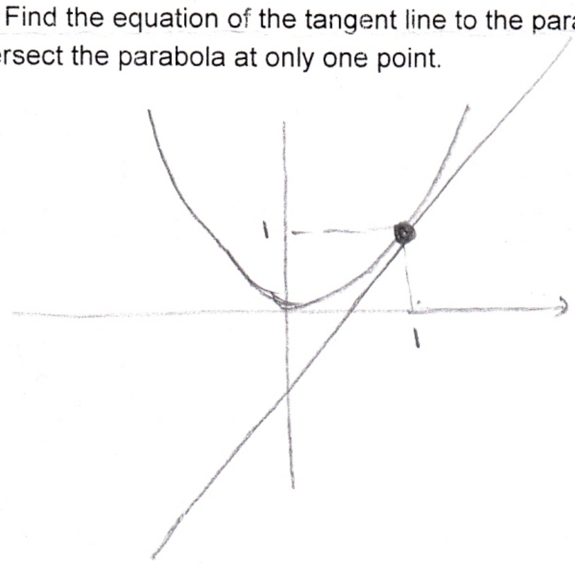
$D_{f\circ g} = (-\infty, 1]$

$R_{f\circ g} = [1, \infty)$



[2] Sketch the graph of the composition $f\circ g$ on the grid provided.

19. Find the equation of the tangent line to the parabola $y=x^2$ at the point $(1,1)$. The tangent lines are the lines that intersect the parabola at only one point. [T/I 3 marks]



$\begin{cases} y=x^2 \\ y=mx+b \\ P(1,1) \end{cases} \Rightarrow \begin{cases} y=x^2 \\ 1=m+b \\ y=mx+b \end{cases} \Rightarrow$

$b=1-m$

$\begin{cases} y=x^2 \\ y=mx+1-m \end{cases} \Rightarrow x^2=mx+1-m \Rightarrow x^2-mx+m-1=0$

Tangent line has an unique intersection with parabola

$\begin{cases} \Delta=0 \\ \Delta=(-m)^2-4(1)(m-1) \end{cases}$

$m^2-4m+4=0$

$(m-2)^2=0$

$m=2$

$\therefore\ y=2x+(1-2)$

$y=2x-1$ (the equation of the tangent line)