MATHEMATICS GRADE 12 MHF4U, ADVANCED FUNCTIONS **TEST Chapter 5** **EXPONENTIAL AND LOGARITHMIC FUNCTIONS** Teacher: Mr Gugoiu	Date WINTER 2009 Name	$\frac{\quad}{50}$ =

Questions 1-5 are Multiple-Choice Questions [K/U 1 mark each]

1. The expression $0.125 = 2^{-3}$ is equivalent to:

A) $3 = \log_2 0.125$ B) $-3 = \log_{0.125} 2$ C) $2 = \log_3 0.125$

D) $-3 = \log_2 0.125$ E) $-0.125 = \log_2 3$

2. The exact value of $\log_{\sqrt{2}} \sqrt[5]{8}$ is: $= \frac{\log 2^{3/5}}{\log 2^{1/2}} = \frac{3}{5} \cdot \frac{2}{1} = \frac{6}{5}$

A) $5/8$ B) 3 C) $6/5$ D) $8/5$ E) 8

3. The solution of the equation $\log \frac{1}{x} = -3$ is: $10^{-3} = \frac{1}{x}$ $x = 10^3$

A) -30 B) 1000 C) $1/1000$ D) $-1/1000$ E) 100

4. If $\log_a(2a-1) = 1$ then: $a = 2a - 1$

A) $a = 0$ B) $a = 1$ C) $a = -1$ D) $a = 2$ E) $a = -2$

5. The approximate value of $\log_{0.1} 0.2$ is:

A) 0.20 B) 0.10 C) 0.70 D) -0.20 E) -0.10

Questions 6-10 are True-False Questions [K/U 1 mark each]

6. The logarithmic function $f(x) = \log_a x$ is always increasing. T (F)

7. The exponential function $f(x) = a^x$ has the vertical asymptote $x = 0$. T (F)

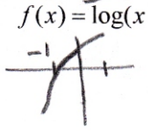
8. The expression $0^1 = 0$ is equivalent to $1 = \log_0 0$. T (F)

9. One key point on the graph of $y = a^x$ is $(-1, 1/a)$. (T) F

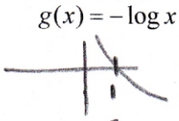
10. The product law of logarithms states that $\log_a(xy) = (\log_a x)(\log_a y)$. T (F)

11. Match the functions from the left side with a graph from the right side. Some functions have no corresponding graph. [A 4 marks]

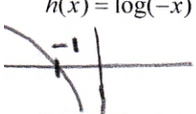
A) [III] $f(x) = \log(x+2)$



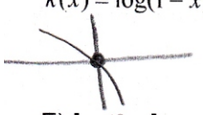
B) [ ] $g(x) = -\log x$



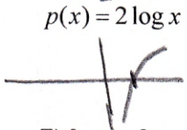
C) [IV] $h(x) = \log(-x)$



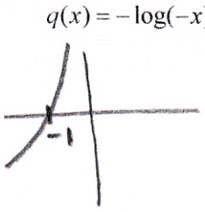
D) [II] $k(x) = \log(1-x)$

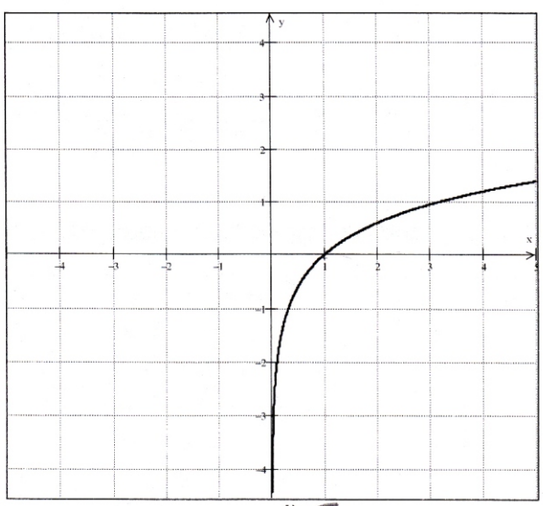


E) [I] $p(x) = 2\log x$

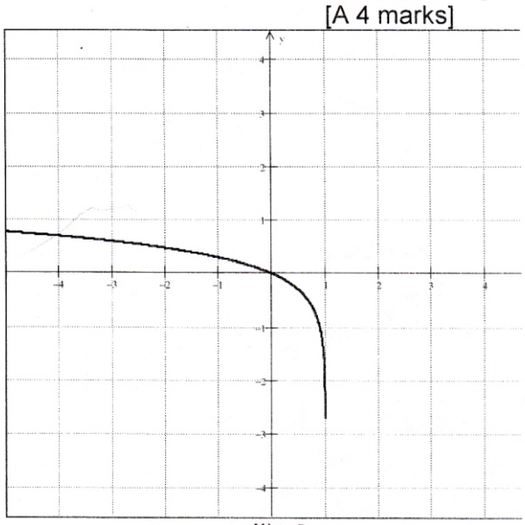


F) [ ] $q(x) = -\log(-x)$

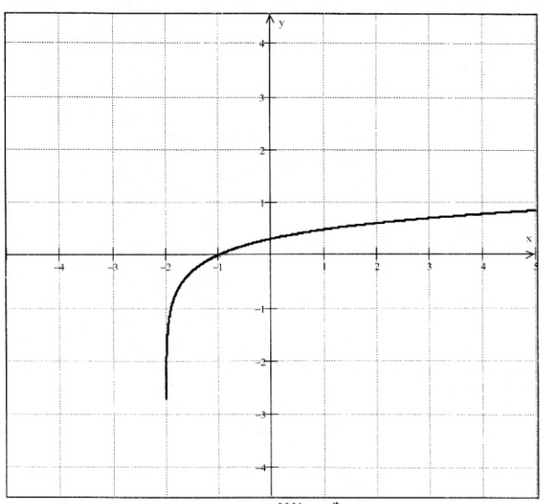




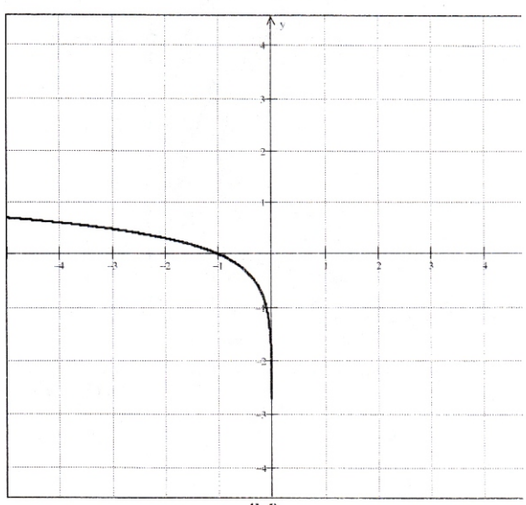
I) E



II) D



III) A



IV) C

The following questions are long answer questions. Show your work to get full marks.

12. [K/U 2 marks]

a) Convert the exponential notation into the logarithmic notation.

$4^{3/2} = 8$

$\frac{3}{2} = \log_4 8$

a) Convert the logarithmic notation into the exponential notation.

$\log 0.001 = -3$

$10^{-3} = 0.001$

13. Solve for x. [K/U 6 marks]

[1] a) $3^{2x} = 81$

$3^{2x} = 3^4$

$2x = 4$

$\therefore x = 2$

[1] b) $\log(x+1) = -1$ $\quad x > -1$

$10^{-1} = x+1$

$x = \frac{1}{10} - 1$

$\therefore x = -\frac{9}{10}$; $x > -1$

[2] c) $2^{x^2-3} = 0.25$

$2^{x^2-3} = \frac{1}{4} = 2^{-2}$

$x^2 - 3 = -2$

$x^2 = 1$

$\therefore x = \pm 1$

[2] d) $\ln x + \ln(x-1) = 0$; $x > 0$; $x > 1$

$\ln x(x-1) = \ln 1$

$x^2 - x = 1$

$x^2 - x - 1 = 0$

$x = \frac{1 \pm \sqrt{1+4}}{2}$

$x = \frac{1-\sqrt{5}}{2} < 0$; $x = \frac{1+\sqrt{5}}{2} > 1$

$\therefore x = \frac{1+\sqrt{5}}{2} \simeq 1.62$

14. Simplify. State any restrictions. $E = 2\log w + 3\log\sqrt{w} + \frac{1}{2}\log w^2$ [A 3 marks]

$w > 0$

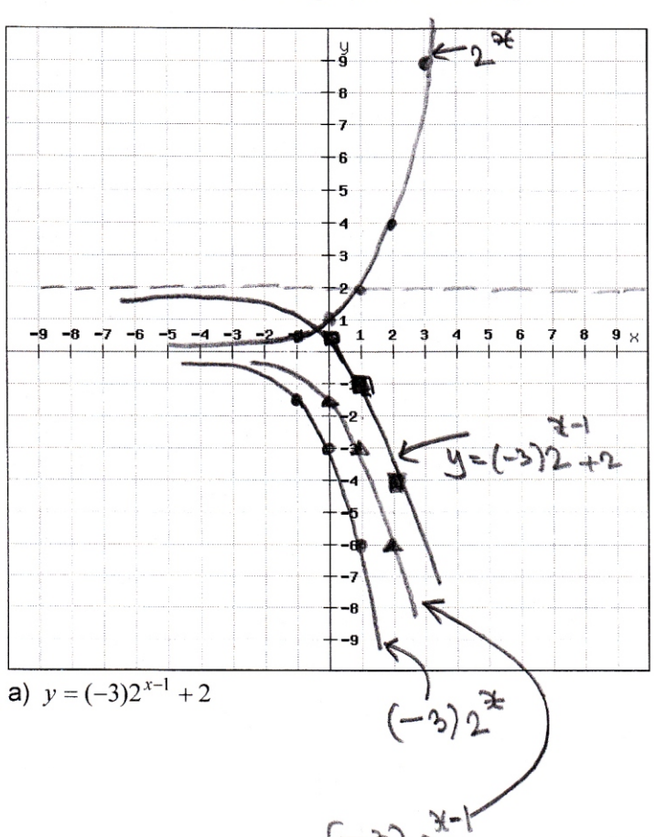
$\log w^2 + \log w^{3/2} + \log w^{\frac{2}{2}}$

$= \log(w^2 \cdot w^{3/2} \cdot w)$

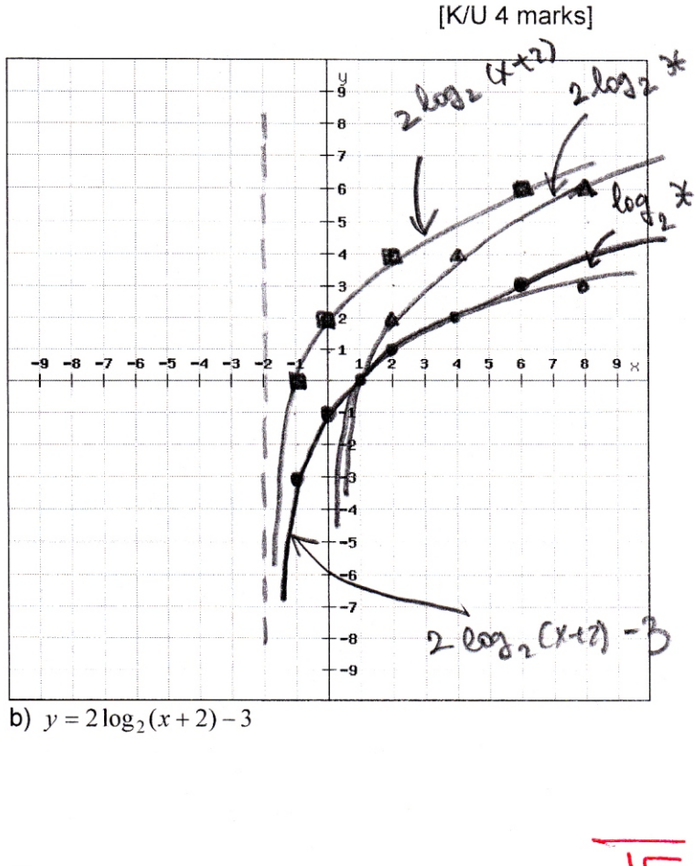
$= \log(w^{3+\frac{3}{2}}) = \frac{9}{2}\log w \Rightarrow$

$\therefore E = \frac{9}{2}\log w$; $w > 0$

15. Use transformations to graph the following functions. [K/U 4 marks]



a) $y = (-3)2^{x-1} + 2$



b) $y = 2\log_2(x+2) - 3$

16. Solve for x. [K/U 4 marks]

[2] a) $2^{x-1} \geq 0.25$

$2^{x-1} \geq 2^{-2}$

$x-1 \geq -2$

$\therefore x \geq -1$

[2] a) $\log_{0.1}(x-1) \leq -1$ $x > 1$

$\log_{0.1}(x-1) \leq -\log_{0.1} 0.1$

$\log_{0.1}(x-1) \leq \log_{0.1}(0.1)^{-1}$

$x-1 \geq (0.1)^{-1}$

$x \geq 1 + \frac{1}{0.1}$

$\therefore x \geq 11$

17. Solve for x: [A 6 marks]

[3] a) $2^x - 2^{-x} = 4$

$2^x = y$

$y - \frac{1}{y} = 4$

$y^2 - 4y - 1 = 0$

$y = \frac{4 \pm \sqrt{16+4}}{2} = 2 \pm \sqrt{5}$ ($2+\sqrt{5} > 0$, $2-\sqrt{5} < 0$)

$2^x = 2 + \sqrt{5}$

$x \ln 2 = \ln(2+\sqrt{5})$

$\therefore x = \frac{\ln(2+\sqrt{5})}{\ln 2} \simeq 2.083$

[3] a) $\log_2(x+5) - \log_2(2x) = 3$

$x > -5$ & $x > 0$

$\log_2 \frac{x+5}{2x} = \log_2 8$

$\frac{x+5}{2x} = 8$

$x + 5 = 16x$

$5 = 15x$

$\therefore x = \frac{1}{3}$

18. Find the domain, ~~range~~, x- and y-intercept for the following functions. Do not graph. [A 6 marks]

[3] a) $f(x) = \frac{3^x + 3^{-x}}{3^x - 3^{-x}}$

$3^x - 3^{-x} \neq 0 \Rightarrow x \neq 0$

$D = \{x \in \mathbb{R} \mid x \neq 0\}$

y-int $= f(0) \Rightarrow$ not defined

$f(x) = 0 \Rightarrow 3^x + 3^{-x} = 0$ (no solution)

$\therefore$ no x-int

$\therefore$ no y-int

~~$R = (-\infty, -1) \cup (1, \infty)$~~

[3] a) $g(x) = 1 - 2\log\frac{x-1}{x+1}$

$\frac{x-1}{x+1} > 0$

x	-1	1
$x-1$	$- \; - \; -$	$0 \; + \; + \; +$
$x+1$	$- \; - \; 0 \; + \; +$	$+ \; + \; +$
$\frac{x-1}{x+1}$	$+ \; \mid$	$- \; 0 \; +$

$D = (-\infty, -1) \cup (1, \infty)$

$g(0) = 1 - 2\log(-1)$ not defined

$\therefore$ no y-int

$g(x) = 0 \Rightarrow 1 = 2\log\frac{x-1}{x+1}$

$\sqrt{10} = \frac{x-1}{x+1} \Rightarrow \sqrt{10}\,x + \sqrt{10} = x - 1$

x-int $= \frac{-1-\sqrt{10}}{\sqrt{10}-1} \simeq -1.93$

19. A computer, originally purchased for $\$2000$, loses value according to the exponential function $V = 2000\left(\frac{1}{2}\right)^{\frac{t}{H}}$, where V is the value, in dollars, of the computer at any time, t, in years, after purchase and H represents the half-life, in years, of the value of the computer. After one year, the computer has a value of approximately $\$1516$.

[A 5 marks]

[1] a) What is the half-life H of the value of the computer?

$1516 = 2000\left(\frac{1}{2}\right)^{\frac{1}{H}}$

$\frac{1516}{2000} = \left(\frac{1}{2}\right)^{\frac{1}{H}}$

$\frac{1}{H} \cdot \ln\frac{1}{2} = \ln\frac{1516}{2000}$

$H = \dfrac{\ln\frac{1}{2}}{\ln\frac{1516}{2000}} \simeq 2.5$ years

[1] b) What is the value of the computer after three years?

$V = 2000\left(\frac{1}{2}\right)^{\frac{3}{2.5}} \simeq \$870.55$

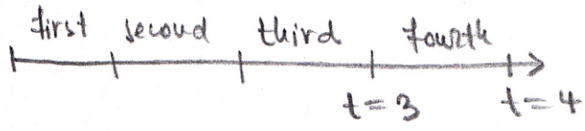
[1] c) How long will it take for the computer to be worth 10% of its purchase price?

$(10\%)\,2000 = 2000\left(\frac{1}{2}\right)^{\frac{t}{2.5}}$

$\ln 0.1 = \frac{t}{2.5}\ln 0.5$

$t = 2.5\,\dfrac{\ln 0.1}{\ln 0.5} \simeq 8.3$ years

[2] How much will be the depreciation of the value of the computer during the fourth year?



$\text{Depreciation} = V(3) - V(4)$

$= 2000\left[\left(\frac{1}{2}\right)^{\frac{3}{2.5}} - \left(\frac{1}{2}\right)^{\frac{4}{2.5}}\right]$

$\simeq \$210.8$