MATHEMATICS GRADE 12
MHF4U, ADVANCED FUNCTIONS

FALL 2008

Date .....

TEST Chapter 1
INEQUATIONS, EQUATIONS, AND GRAPHS
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Name SOLUTIONS

$\frac{\quad}{50}$ =

Questions 1-5 are Multiple-Choice questions [K/U 1 mark each]

1. Which of the following relations is NOT true?

A) $|a-b|=|b-a|$ B) $\sqrt{a^2}=|a|$ C) $|a|\geq 0$ (D) $|-2a|<2|-a|$ E) $|a-b|\leq|a|+|b|$

2. The interval notation for $-3\leq x<2$ is;

A) $(-3,2)$ B) $[-3,2]$ C) $[-3,3)$ D) $[-2,3)$ (E) $[-3,2)$

3. The inequality $x^2\geq 16$ is equivalent (has the same solution set) with:

A) $|x|\geq 16$ (B) $|x|\geq 4$ C) $|x|\leq 4$ D) $|x|\leq 16$ E) $|x|\geq 8$

4. The distance between the point $A(2,-3)$ and $B(-6,+3)$ is equal to:

A) $\sqrt{10}$ B) 100 C) 5 D) 20 (E) 10

5. The y-intercept point(s) of the relation $\frac{x^2}{4}+\frac{y^2}{9}=1$ is (are):

A) $(-3,0)$ *and* $(3,0)$ B) $(0,3)$ *and* $(3,0)$ (C) $(0,-3)$ *and* $(0,3)$ D) $(-3,0)$ E) $(3,0)$

Questions 6-10 are True-False questions [K/U 1 mark each]

6. If the point $P(a,b)$ is in quadrant II, then the point $Q(-a,-\frac{1}{b})$ is in quadrant IV. (T) F

7. A graph is symmetric with respect to the origin O, if whenever $P(x,y)$ is a point on the graph, $P(x,-y)$ is also a point on the graph. T (F)

8. There is a correspondence one-to-one between the ordered pairs (a,b) and the points on a plane. (T) F

9. The relation $xy^3=1$ is symmetric with respect to the origin O. (T) F

10. The un-bounded interval $[2,\infty)$ is open always on the right because ∞ is not considered a number. (T) F

11. Match the relations from the left side with a graph from the right side. Some functions may have no corresponding graph. [A 4 marks]

A) $|x-1|=2$ X

B) $x^2-4=5$ (I)

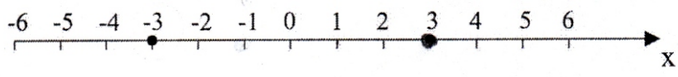
C) $|x|>2$ X

(II) D) $(x-2)(x-5)<0$ 2 5

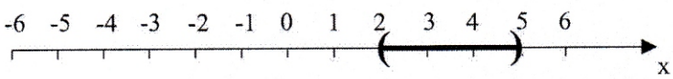
E) $\frac{1}{x}<1$ (IV)

F) $\sqrt{x}>2$ (III)

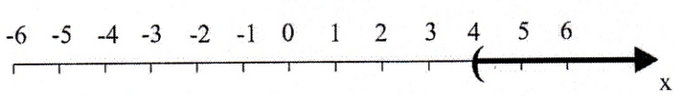
I) B



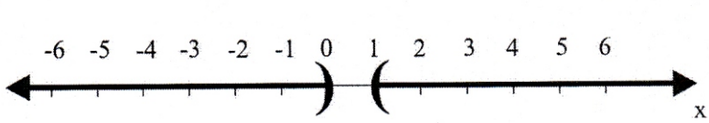
II) D



III) F



IV) E

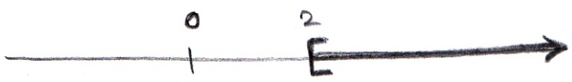


12. Solve each inequality and then graph it. [K/U] [4 marks]

a) $2-3x \le -8+2x$

$10 \le 5x$

$\therefore\ x \ge 2$



b) $1+2(x-3) > -3(1-2x)+4$

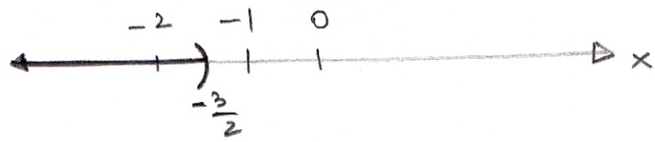
$1+2x-6 > -3+6x+4$

$-5-1 > 4x$

$-6 > 4x$

$x < -\frac{6}{4}$

$\therefore\ x < -\frac{3}{2}$



13. Solve each equality or inequality and then graph it. [K/U] [4 marks]

a) $|3-2x| = 6$

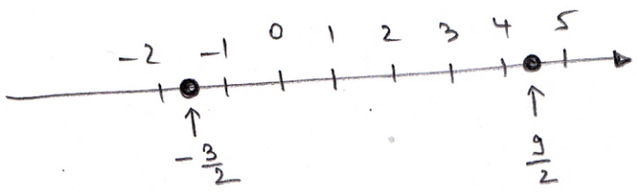
$3-2x = \pm 6$

$2x = 3 \mp 6$

$x = \frac{3-6}{2}$ or $x = \frac{3+6}{2}$

$\therefore\ x = -\frac{3}{2}$ or $x = \frac{9}{2}$

$\therefore\ x \in \{-\frac{3}{2}, \frac{9}{2}\}$



b) $|2x-1| \le 2$

$-2 \le 2x-1 \le 2$

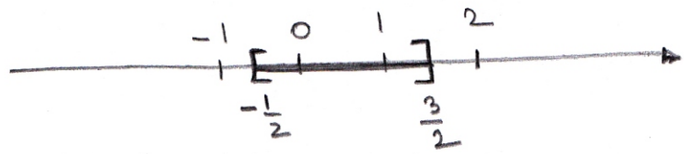
$-2 \le 2x-1$ and $2x-1 \le 2$

$x \ge \frac{-2+1}{2}$ and $x \le \frac{2+1}{2}$

$x \ge -\frac{1}{2}$ and $x \le \frac{3}{2}$

$\therefore\ -\frac{1}{2} \le x \le \frac{3}{2}$

$x \in [-\frac{1}{2}, \frac{3}{2}]$



14. Write the equation of the circle represented in the right figure. [K/U] [2 marks]

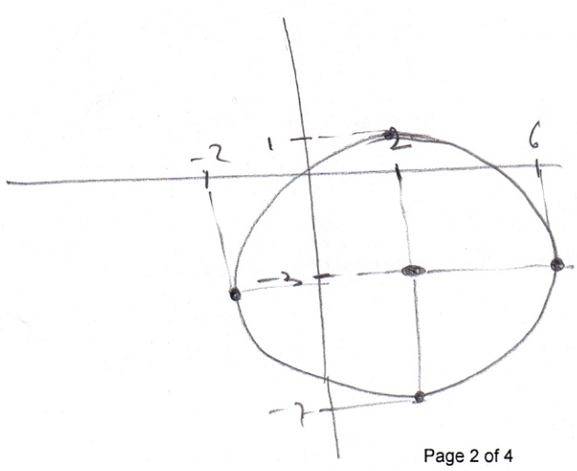
$x_c = 2$

$y_c = -3$

$R = 4$

$(x-x_c)^2 + (y-y_c)^2 = R^2$

$\therefore\ (x-2)^2 + (y+3)^2 = 16$



15. Analyse the symmetry of each equation. Do not graph. [A] [6 marks]

a) $xy^2 = 1$

$(-x)y^2 = 1$ (different)

$x(-y)^2 = 1$ (same)

∴ [symmetric with respect to the x-axis]

$(-x)(-y)^2 = 1$ (different)

∴ no symmetry with respect to the origin

∴ no symmetry in the y-axis

b) $xy = \frac{1}{x} + \frac{1}{y}$

$(-x)y = \frac{1}{-x} + \frac{1}{y}$ (different)

$x(-y) = \frac{1}{x} + \frac{1}{-y}$ (different)

$(-x)(-y) = \frac{1}{-x} + \frac{1}{-y}$ (different)

∴ [no symmetry]

c) $|x| + |y| = 1$

$|-x| + |y| = 1$ (same)

$|x| + |-y| = 1$ (same)

$|-x| + |-y| = 1$ (same)

∴ The graph is symmetri with respect to

a) x-axis

b) y-axis

c) origin O

16. Graph the following circle on the grid provided on the right. [A] [4 marks]

$x(x+2) + y^2 = 3(2y+2)$

$x^2 + 2x + y^2 = 6y + 6$

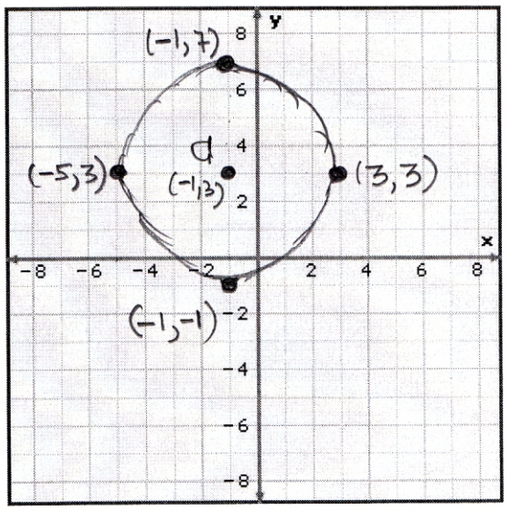
$x^2 + 2x + y^2 - 6y = 6$

$(x+1)^2 - 1 + (y-3)^2 - 9 = 6$

$(x+1)^2 + (y-3)^2 = 16 = 4^2$

∴ C $(-1, 3)$

$R = 4$



17. Solve the inequality and then graph it. Show your work. [A] [4 marks]

$$\frac{-x^2(x+1)^3}{(x-3)^2(x+2)} \geq 0$$

x	$-\infty$		-2		-1		0		3		∞
$-x^2$	−	−	−	−	−	−	0	−	−	−	−
$(x+1)^3$	−	−	−	−	0	+	+	+	+	+	+
$(x-3)^2$	+	+	+	+	+	+	+	+	0	+	+
$x+2$	−	−	0	+	+	+	+	+	+	+	+
	−	−	|	+	+	0	−	−	0	−	−	|	−	−

∴ $-2 < x \leq -1$ or $x = 0$

$x \in (-2, -1] \cup \{0\}$

18. Graph the following semicircle $x = -3 + \sqrt{y(4-y)}$. Show your work. [A] [4 marks]

$x + 3 = \sqrt{y(4-y)}$

$(x+3)^2 = y(4-y)$

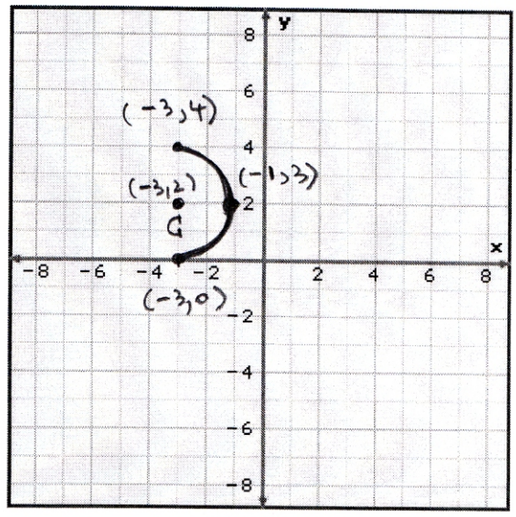
$(x+3)^2 = 4y - y^2$

$(x+3)^2 + y^2 - 4y = 0$

$(x+3)^2 + (y-2)^2 = 2^2$

$\therefore C(-3, 2)$

$R = 2$



19. Solve the inequality and then graph it. Show your work. [A] [5 marks]

$\frac{4}{x} + \frac{3}{x+1} > \frac{3}{x-1}$

$\frac{4}{x} + \frac{3}{x+1} - \frac{3}{x-1} > 0$ [1]

$\frac{4(x^2-1) + 3(x^2-x) - 3(x^2+x)}{x(x+1)(x-1)} > 0$

$\frac{4x^2 - 4 + 3x^2 - 3x - 3x^2 - 3x}{x(x+1)(x-1)} > 0$

$\frac{4x^2 - 6x - 4}{x(x+1)(x-1)} > 0$ [1]

$\frac{2(2x^2 - 3x - 2)}{x(x+1)(x-1)} > 0$

$\frac{2(2x+1)(x-2)}{x(x+1)(x-1)} > 0$ [1]

x	$-\infty$	-1	$-\frac{1}{2}$	0	1	2	∞
$2x+1$	$-$	$-$	0	$+$	$+$	$+$	$+$
$x-2$	$-$	$-$	$-$	$-$	$-$	0	$+$
x	$-$	$-$	$-$	0	$+$	$+$	$+$
$x+1$	$-$	0	$+$	$+$	$+$	$+$	$+$
$x-1$	$-$	$-$	$-$	$-$	0	$+$	$+$
All	$-$	0	$+$	0	$-$	0	$+$

[1]

$\therefore x \in (-1, -\frac{1}{2}) \cup (0, 1) \cup (2, \infty)$ [1]

20. Graph the following relation $|x - y| = 2$
Show your work (give reasons why your answer is that.) [T/I] [3 marks]

$|x - y| = 2$

$x - y = \pm 2$

$y = x \mp 2$

$y = x - 2$ or $y = x + 2$